EXTRAGALACTIC VARIABLE RADIO SOURCES

by

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ABSTRACT

The standard model for extragalactic variable radio sources comprises an isotropically expanding plasmoid with "frozen" magnetic flux and an electron distribution which evolves adiabatically. This model leads to the following relation between the peak luminosity $L_{\nu,m}$ and the relevant frequency ν_m, , which are functions of time: $L_{\nu,m} \propto \nu_m^{\ N}$ where $N = (7n + 5)/(4n + 5)$. In this expression, n is the spectral index in the optically thin part of the spectrum, where $L_\nu \propto \nu^{-n}$. For n in the range 0.5 to 1.5, the standard model yields N in the range 1.2 to 1.4. By contrast, analysis of observational data yields estimates of N in a small range about the mean value 0.4, in clear contradiction with the standard model.

The model is here modified to comprise finite flux tubes, either rooted in a parent object or forming closed toroids. The expansion rate along the flux tube may differ from the expansion rate transverse to the flux tube. The electron distribution is assumed to remain isotropic and to evolve adiabatically. This model yields a good match to the above observational relation in the case that the flux tubes expand along their length but not in the transverse direction.

Subject Headings: radio sources: galaxies - quasars - radio emission

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I. Introduction

It has been known, since the observations of Dent (1965), that radio emission in the millimeter and centimeter range from quasars and some radio galaxies is variable on a time scale of years, months, and sometimes shorter intervals. Soon after these variations were discovered, it was suggestd by Shklovsky (1965) that the varying components are sources of synchrotron radiation which are initially optically thick but, as they expand, become optically thin at progressively longer wave lengths.

This proposal was analyzed in more detail by van der Laan (1966), who analyzed the properties of a model comprising a uniform and spherical cloud of magnetic field and relativistic electrons. The electron distribution was assumed to be isotropic, with a power-law dependence on energy. The magnetic field was assumed to be "frozen" in the cloud so that magnetic flux is conserved. The expansion was assumed to be spherically symmetric so that there is no change of topology of the magnetic field and the electron distribution remains isotropic. A further assumption was that the relativistic gas cools adiabatically during expansion.

This model was compared with observational data by Kellermann and Pauliny-Toth in 1968, and the agreement was quite satisfactory. However, the situation has changed in recent years.

Andrew, MacLeod, Harvey and Medd (1978) have analyzed results of a ten year study of extragalactic variable sources at centimeter wavelengths. They examined 99 variable sources and found that bursts of particle acceleration

occur in the average variable source about once every one and one-half to two years.

At any time in the evolution of the source, the flux (per unit frequency) F_ν has a maximum value $F_{\nu,m}(t)$ at a frequency $\nu_m(t)$. Andrew et al. (1978) found that, for any source, these quantities vary in such a way that they satisfy approximately the relation

$$F_{\nu,m}(t) \propto [\nu_m(t)]^N \ . \tag{1.1}$$

They found that observationally determined values of N fall in a narrow range centered on 0.4:

$$N = 0.4^{+0.15}_{-0.2} \ . \tag{1.2}$$

By performing a similar analysis of data compiled by Altschuler and Wardle (1976, 1977), Andrew et al. (1978) determined N values in the range $N = 0.4 \pm 0.15$.

According to the SVDL (Shklovsky-van-der-Laan model), N should be related to n, the power-law index of the radio emission, by the relation

$$N = \frac{7n + 5}{4n + 5} \ . \tag{1.3}$$

Hence, for $n = 0.5, 0.75, 1.0, 1.5$, we expect that $N = 1.21, 1.28, 1.33, 1.41$, respectively. It is clear that this property of the SVDL model is incompatible with the observational data summarized by Andrew et al. (1978).

II. Flux-tube Model

Since the properties of the SVDL model are broadly consistent with observational data, it seems reasonable to explore minor modifications of this basic model. The simplest variant which came to mind is that the expansion is anisotropic with respect to the magnetic field. This is the possibility which will be explored in this article.

If one is to consider the possibility that the expansion is anisotropic with respect to the magnetic field, it is clear that one should also consider the possibility that the electron distribution is anisotropic. Discussion of this possibility is deferred for a subsequent article. For the time being, we consider that the electron distribution remains isotropic, due either to particle-particle scattering or to scattering by magnetic fluctuations, or to some other process.

We consider a single magnetic flux tube of uniform radius R and length L. The flux tube may be rooted in an accretion disk, in which case it is to be assumed that the detailed geometry is such as to provide mirror action at the ends of the tube (Fig. 1(a)). Alternatively, the flux tube may be in the form of a toroid, in which case L measures the perimeter (Fig. 1(b)). As an additional simplification, we assume that the transverse component of the magnetic field (transverse to the major direction measured by L) is small compared with the longitudinal component of the magnetic field (in the direction measured by L).

We assume that L and R vary with time t according to relations

$$L = L_1 t^{\lambda}, \; R = R_1 t^{\rho} \qquad (2.1)$$

where $f(t)$ is an arbitrary function of time and L_1, R_1, etc., are the values of L, R, etc., at $t = t_1$, assuming that $f(t_1) = 1$. For unaccelerated expansion, $f(t) = t/t_1$. For any reasonable assumption concerning the conductivity of the plasma, we find that the total flux of the tube is constant so that the magnetic field strength B varies as follows,

$$B = B_1 f^{-2\rho} . \tag{2.2}$$

For a relativistic gas with an isotropic pressure p, the pressure varies with density n according to the relation

$$\rho \propto n^{4/3} . \tag{2.3}$$

The pressure of electrons with energy in a small range about the value E (eV), is proportional to nE. Hence, noting that

$$n = n_1 f^{-(\lambda+2\rho)} , \tag{2.4}$$

we see that

$$E \propto f^{-(1/3)(\lambda+2\rho)} . \tag{2.5}$$

We suppose that the total number of relativistic electrons in the plasmoid has a power-law distribution of the form

$$dN = KE^{-m}dE . \tag{2.6}$$

Since the total number of electrons is fixed, and since the evolution of energy with time is taken to be the adiabatic form (2.5), we find that

$$K(t) = K_1 f^{-(1/3)(\lambda+2\rho)(m-1)} . \tag{2.7}$$

If the lower and upper limits of the power-law spectrum (2.6) are taken to be $E_L(t)$ and $E_U(t)$, respectively, we see from Eq. (2.5) that

$$E_L = E_{L,1} f^{-(1/3)(\lambda+2\rho)}, \; E_U = E_{U,1} f^{-(1/3)(\lambda+2\rho)} . \tag{2.8}$$

One may verify from (2.6), (2.7) and (2.8) that the total number of electrons is fixed, being given by

$$N_T = \frac{K_1}{m-1} [E_{L,1}^{-(m-1)} - E_{U,1}^{-(m-1)}] \approx \frac{K_1}{m-1} E_{L,1}^{-(m-1)} . \tag{2.9}$$

III. Radiation

We now consider the synchrotron radiation produced by the electron distribution proposed in § II. For simplicity, we represent the radiation spectrum of a single electron S_1(erg $s^{-1}Hz^{-1}$) by the delta-function approximation

$$S_{1,\nu} = 2A_1 E^2 B^2 \delta(\nu-\nu_p), \; A_1 \approx 10^{-26.1} \tag{3.1}$$

where

$$\nu_p = A_2 E^2 B, \quad A_2 \approx 10^{-5.3}. \tag{3.2}$$

On combining these equations with Eq. (2.6), we obtain the standard result that the total luminosity spectrum $L_\nu(\text{erg s}^{-1}\text{Hz}^{-1})$, defined by

$$L_\nu = \int dN\, S_{1,\nu} , \tag{3.3}$$

is given by

$$L_\nu = A_1 A_2^{n-1} K B^{n+1} \nu^{-n} , \tag{3.4}$$

where

$$n = \frac{1}{2}(m - 1) . \tag{3.5}$$

The lower and upper bounds of this power-law spectrum are given by

$$\nu_L = A_2 E_L^2 B, \quad \nu_U = A_2 E_u^2 B . \tag{3.6}$$

Following van der Laan (1966) and Scheuer (1967), we estimate the radiation in the self-absorbed part of the spectrum by noting that the brightness temperature cannot exceed the equivalent temperature of electrons chiefly responsible for the synchrotron radiation at any given frequency. On using the Rayleigh-Jeans formula for the self-absorbed form of the surface emissivity $F_{\nu,SA}$ (erg cm^{-2}s^{-1}Hz^{-1}) ,

$$F_{\nu,SA} = A_3 E\nu^2, \quad A_3 \approx 10^{-32.0} , \tag{3.7}$$

and using Eq. (3.2) to express E in terms of ν, we obtain the relation

$$F_{\nu,SA} = A_4 B^{-1/2}\nu^{5/2}, \quad A_4 \approx 10^{-30.3} . \tag{3.8}$$

If L and R remain comparable in magnitude for all time, we are effectively considering isotropic expansion, that is, we are reconsidering the SVDL model. It is not possible that L should be much less than R. Hence, we consider only the case that L >> R. With this inequality, the effective area of the plasmoid is effectively $2\pi RL$, so that the self-absorbed part of the luminosity spectrum is given by

$$L_{\nu,SA} = A_5 LRB^{-1/2}\nu^{5/2}, \quad A_5 \approx 10^{-29.5} . \tag{3.9}$$

We assume that the actual spectrum is given by (3.9) up to the frequency ν_m for which expressions (3.4) and (3.9) are equal, and by expression (3.4) for values of ν above ν_m. We then find that the frequency ν_m of peak luminosity is given by

$$\nu_m = [A_1 A_2^{n-1} A_5^{-1} K\, L^{-1} R^{-1} B^{n+3/2}]^{\frac{1}{n+5/2}} , \tag{3.10}$$

and the peak luminosity $L_{\nu,m}$ is given by

$$L_{\nu,m} = [A_1^{5/2} A_2^{(5/2)(n-1)} A_5^n K^{5/2} L^n R^n B^{2n+5/2}]^{\frac{1}{n+5/2}} . \tag{3.11}$$

On using Eqs. (2.1), (2.2), (2.7) and (3.5), we find that $\nu_m(t)$ and $L_{\nu,m}(t)$ depend on time, through the function f(t), as follows:

$$\nu_m = \nu_{m,1} f^{\frac{-(2/3)n+1}{n+5/2}\lambda - \frac{(10/3)n+4}{n+5/2}\rho} , \tag{3.12}$$

and

$$L_{\nu,m} = L_{\nu,m,1} f^{-\frac{(2/3)n}{n+5/2}\lambda - \frac{(19/3)n+5}{n+5/2}\rho} , \tag{3.13}$$

where

$$\nu_{m,1} = [A_1 A_2^{n-1} A_5^{-1} K_1 L_1^{-1} B_1^{n+3/2}]^{\frac{1}{n+5/2}} , \tag{3.14}$$

and

$$L_{\nu,m,1} = [A_1^{5/2} A_2^{(5/2)(n-1)} A_5^n K_2^{5/2} L_1^n R_1^n B_1^{2n+5/2}]^{\frac{1}{n+5/2}} . \tag{3.15}$$

We see from the above equations that there should be a power-law relation between $L_{\nu,m}$ and ν_m, which we write as

$$L_{\nu,m}(t) \propto [\nu_m(t)]^N , \tag{3.16}$$

where

$$N = N_{\lambda,\rho} \equiv \frac{2n\lambda + (19n + 15)\rho}{(2n + 3)\lambda + (10n + 12)\rho} \tag{3.17}$$

IV. Discussion

We may readily verify that the case $\lambda = 1, \rho = 1$, corresponding to isotropic expansion, yields

$$N_{1,1} = \frac{7n + 5}{4n + 5} \quad . \tag{4.1}$$

This reproduces the familiar relationship between $L_{\nu,m}$, and ν_m for the SVDL model (van der Laan, 1966).

We now consider two other special cases. The case $\lambda = 0, \rho = 1$ corresponds to a toroidal plasma for which the perimeter is fixed but the minor radius grows progressively. It is clear that such a model would eventually break down, since the radius would at some stage exceed the perimeter. Nevertheless, we note that this model yields the following value of N:

$$N_{0,1} = \frac{19n + 15}{10n + 12} \quad . \tag{4.2}$$

We also consider the case that $\lambda = 1, \rho = 0$, corresponding to toroidal plasma of constant cross section, which increases progressively in length

$$N_{1,0} = \frac{2n}{2n + 3} \quad . \tag{4.3}$$

We list the values of the above formulas for a range of values of n in Table 1. It is clear that the second case, $N_{0,1}$, provides an even worse fit to observational data than does the SVDL model, $N_{1,1}$. On the other hand, the third model, characterized by $N_{1,0}$, appears to provide a quite acceptable fit to observational data. We see that for N in the range $0.65 < n < 1.50$, N is in the range $0.3 < N < 0.5$.

It is clearly of interest to pursue the case that $\lambda = 1$, $\rho = 0$. For this case, Eqs. (3.12) and (3.13) yield the following dependence of ν_m and $L_{\nu,m}$ on f(t):

$$\nu_m = \nu_{m,1} f^{-\Gamma} , \tag{4.4}$$

and

$$L_{\nu,m} = L_{\nu,m,1} f^{-\Lambda} \tag{4.5}$$

where

$$\Gamma = \frac{4n + 6}{6n + 15} \tag{4.6}$$

and

$$\Lambda = \frac{4n}{6n + 15} . \tag{4.7}$$

Values of these indices are given, for a range of values of n, in Table 2. We see that Λ varies only over the range 0.44 - 0.52 as n covers the range 0.5 - 2.0, whereas Λ varies from 0.11 - 0.30 over the same range of values of n.

Clearly, one cannot infer from these relations the expected dependence of ν_m and $L_{\nu,m}$ on t unless one can determine the expected dependence of f upon

t. If it can be argued that L increases linearly with time, so that $f(t) \propto t$ then we see from Table 2 that, to fair approximation, $\nu_m \propto t^{1/2}$.

As tests of the proposed model, it is desirable to examine cases for which individual bursts are well identified to try to check the dependence of N upon n, given by Table 1, and the dependence of ν_m and $L_{\nu,m}$ on t, given by Table 2.

Finally, we inquire into mechanisms which might produce plasmoids which expand progressively in length, while the cross section remains substantially constant. We first consider the property that the radius of the flux tubes does not change, or changes only slightly, during the evolution of the plasmoid. This is equivalent to the requirement that the mean magnetic field strength remains constant. If the dominant stress in the plasmoid is the magnetic stress, this condition is equivalent to constant pressure, implying that the plasmoid is moving in a constant-pressure medium.

There are at least three processes which might lead to a progressive increase in the length of the flux tube, as indicated schematically in Fig. 2. If the flux tube is rooted in an accretion disk, then differential rotation of the disk may lead to a progressive stretching of the loop, as indicated in Fig. 2(a). If, on the other hand, the plasmoid comprises a toroid ejected from the disk, the steady increase in length may be due to an initial velocity of expansion of the toroid (Fig. 2(b)). This expansion could possibly be due to centrifugal motion, if the formation of the toroid is accomplished by flare action and if, as is the case in solar flares, the flare process leads to the formation of a dense flare plasma in the plasmoid created by the flare. However, the centrifugal force would decrease with time more rapidly than the radially inward force due to magnetic tension, so that the

expansion would ultimately be reversed. If the toroidal plasma is created with a high velocity in the radial (major radius) direction, we again find that magnetic tension would eventually reverse the expansion.

A third possibility which comes to mind is that the plasmoid is turbulent, and that - as is well known - the turbulence leads to progressive stretching of magnetic field lines. If the external pressure is substantially constant, the magnetic field strength will remain constant. For any portion of the total flux tube comprising the plasmoid, the behavior is as described in this article. However, the total geometry is quite different: the overall shape may be spherical and the overall expansion may be spherically symmetric. Hence the model would be similar to the SVDL model except that the magnetic field strength remains constant rather than decreasing with time. This model is currently being examined.

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Table 1

Dependence of N upon n

n	$N_{1,1}$	$N_{0,1}$	$N_{1,0}$
0.5	1.21	2.72	0.25
0.75	1.28	1.5	0.33
1	1.33	1.54	0.4
1.5	1.41	1.61	0.5
2	1.46	1.66	0.57

Table 2

Dependence of Γ and Λ on n

n		
0.5	0.44	0.11
0.75	0.46	0.15
1	0.48	0.19
1.5	0.5	0.25
2	0.52	0.3

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Figure Captions

Fig. 1. (a) Schematic model of flux tube rooted in accretion disk.

(b) Schematic toroidal flux tube ejected from accretion disk.

Fig. 2 (a) Differential rotation in accretion disk leads to stretching of flux tube. (b) Schematic representation of toroidal flux tube, ejected from disk, that expands in major radius but not in minor radius.

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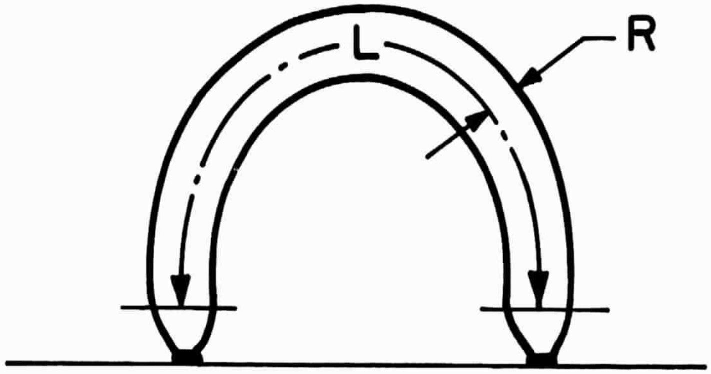
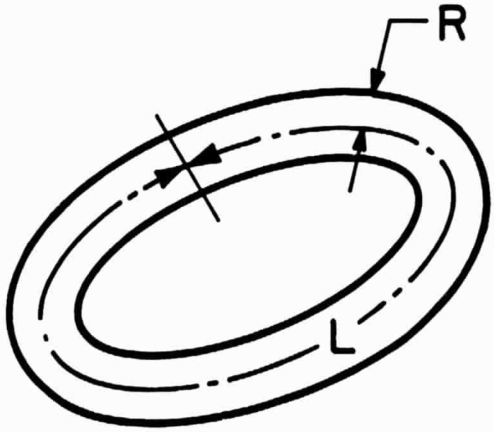


1a



1b

FIGURES
1a & 1b

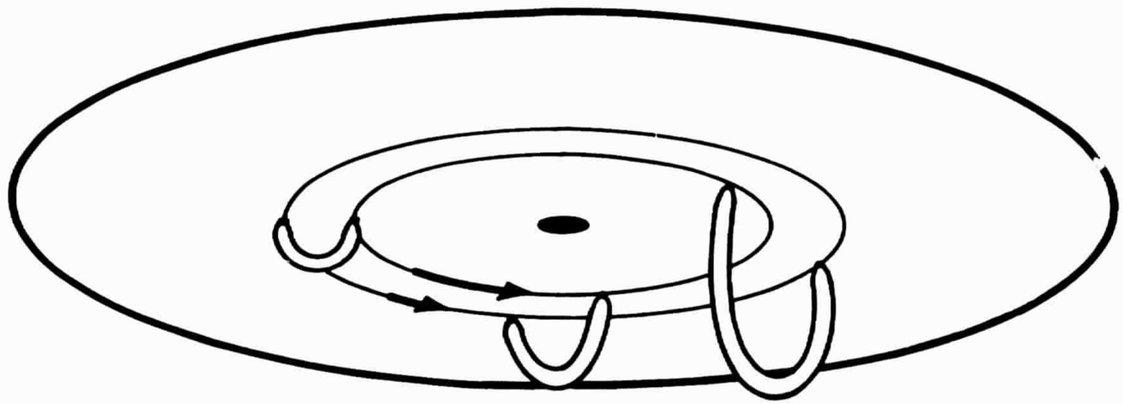


FIGURE
2a

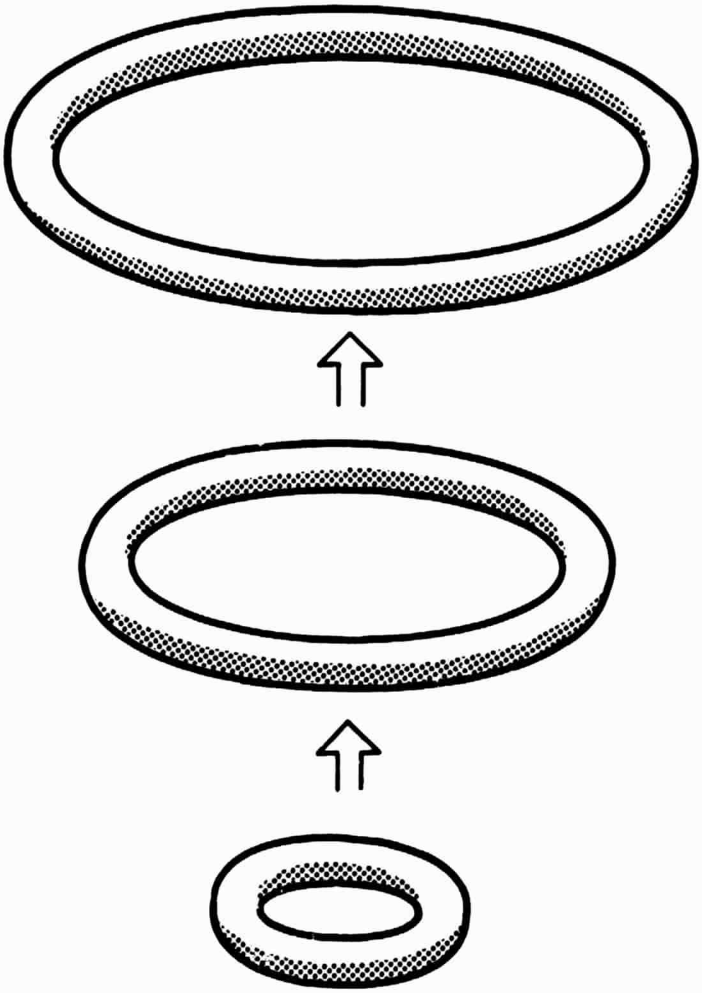


FIGURE
2b